

**BACKGROUND STRUCTURE FUNCTIONS OF THE SEA,  
A BASIS TO REDUCE EMISSIONS AND IMPROVE IMAGES**

by

Michael C. Kobold

A Dissertation Submitted to the Faculty of  
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This dissertation was prepared under the direction of the candidate's dissertation advisor, Dr. Pierre-Philippe J. Beaujean, Department of Ocean and Mechanical Engineering, and has been approved by the members of his supervisory committee. It was submitted to the faculty of the College of Engineering and Computer Science and was accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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## VITA

As a physics student at the University of Texas, Arlington Center for Accelerator Science and Technology, which the Superconducting Super Collider, image and signal processing were dominant tools. As a analysis engineer at General Motors, concurrent work at the University of Michigan helped integrate Microscope imaging systems. While directing repairs for Northrop Grumman (NGC), their radar systems engineer training program in Melbourne, FL, kept him current in remote sensing. At this time he earned a Professional Engineer license from Michigan as a Mechanical Engineer (ME). His ME work included random vibration issues, ranging from simple satellite telescope minimum mode requirements, Mars Explorer Rover shock analysis on the High – Gain Antenna gimbal systems for the long-range communications antenna, to full vehicle systems integration and aircraft sonic fatigue. Moving to Ball Aerospace & Technologies Corp. in Boulder, he analyzed cross-tracking infrared sensors and deformable mirrors for vibration and shock on the Mars Explorer Rovers Spirit and Opportunity for Cal. Inst. of Tech. / NASA Jet Propulsion Lab. That led to a contract to AFRL for remote sensing including structure functions (for WaveTrain<sup>®</sup> and laser vibrometry) and infrared multi – spectral threat warning systems while obtaining a second M.S. (E.E. at AFIT), concentrating in propagation and imaging through turbulence. Sensor fusion and ATR small contracts at AFRL turned into work for NASIC when General Dynamics absorbed Veridian. After a temporary Air Force contract while studying laser engineering at CREOL in Orlando he joined a Navy optics branch of NSWC in Panama City. As a veteran Navy sailor, Mr. Kobold’s engine room experience led to his second US patent of the current suite of four Navy (NSWC) patents, 9197822, 9208386, 11431421, and 11653125.

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In 2017-2018 a Littoral Acoustic Modem Protocol Statistics, sought to provide analysis to help define the challenging task of acoustic communications (aComms) underwater in the littorals. “Distributed Underwater Wireless Sensor Networks” was a Naval Innovation Science and Engineering (NISE) project, at the Naval Surface Warfare Center Panama City Division (NSWC-PCD). Later, NSWC-PCD provided funding via In-house Laboratory Independent Research (ILIR) for “Acoustic Structure Function” projects. The objective was to define, address, and help mitigate deficiencies for aComms in the littoral environment. This work is a partial fulfillment of a Ph.D. program with Florida Atlantic University (FAU) under advisor professor Pierre-Philippe Beaujean. Some preliminary analysis used the Shallow Water Acoustic Toolset (replaced by MASTODON [1]) results for Saint Andrew Bay for posters. Dr. Keith Aliberti created the NISE project. Dr. Frank Crosby is the sponsor for two ILIR Acoustic Structure Function projects that cover this dissertation. The review assistance of all, including Marc Adams, Chuck Bernstein, Drs. Ray Lim, Jesse Angle, Darshan Bryner, and Gary Sammelmann is appreciated, as is other essential work: Kim Tuttle’s efforts to find funding for the final breakthrough, continuing with Ann Marie Shover and Carrie Delcomyn. The SSP data from Daniel Sheahan’s 2007 FAU thesis on UW acoustics of Port Everglades, and the insight of Dr. Bob Headrick (with Dr. Kyle Becker, both of ONR) are much appreciated. Luckily we had reviewers with similar experience in Dr. Joe Lopes and Dr. Matt Bays. The cheerful assistance of NSWC-PCD librarian Adelle Singer was fruitful and helped pave the path laid out by my advisor Dr. Beaujean, one of the few dozen persons knowledgeable in this statistical field.

## ABSTRACT

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Background structure functions (BSFs) are wavefront distortion metrics, functions of sound speed profiles (SSPs) that are functions of depth. Underwater Acoustics (UWA) can use these models for SSP-based forecasting, a form of Matched Field Processing (MFP) that detects signals that are otherwise lost to receivers, to allow communication and imaging with reduced power radiated into the sea. This dissertation plots the UWA distortion for communications (comms) or remote sensing using measured and verified SSPs for 132 different locations in the Atlantic Ocean and near the Caribbean from a NAVO Atlas, as well as 64 SSPs in two areas in the littorals, Saint Andrew Bay and Port Everglades. These *purely* statistical BSF plots use only a few degrees of freedom (DOFs). This design of structure function makes use of related industrial systems – wavefront tracking and control systems, that utilize BSFs. Established UWA methods use Doppler-identified delays that for different deterministic paths using a couple dozen DOFs provide a classical structure function,  $D(1,2)$ . The purely statistical BSFs can forecast approximate  $D(1,2)$  in locations where acoustic measurements are not available, but SSPs are. This dissertation also produces numerical metrics such as phase variance using the *background*, SSPs and

basic bathymetry (depth and range). The results contain phase variance plots for all the locations and tabulation of their overall averages. Forecasts of phase and its variance provides metrics to help phase-locked loop systems maintain lock, improving underwater (UW) acoustic communications (aComms) with, e.g., delay and path-gain models for the use of adaptive filters in equalizers. Avoiding loss of lock also helps lower required transmitted power. A related analysis using BSFs develops a method to help improve images based on sensed wavefront distortion using the percentage of acoustic distortion that is tip/tilt. Finally, an analysis of deep-sea acoustic convergences results in a method to estimate the amount of focusing anomalies and their resulting phase error due to the Gouy phase, a type of Berry phase that is ordinarily analyzed with Catastrophe theory to avoid the breakdown in ray-theory models at UWA focuses.

*Validation of the dissertation and help for the reader:*

**Items** *The quotes,*

*on page 43:*

*“The quantity  $\Phi$  is the rms variation of phase of the signal at the receiver under the geometrical optics approximation... It is important to note that knowledge of the sound-speed fluctuations in the form  $[v(z)]$  is all that is required to determine the values of  $\Phi$  and  $\Lambda$ : no wave-propagation measurements need to be made. We refer to  $\Phi$  as the strength parameter.” [2, p 92]*

*and in § 3.4, on page 127*

*“Figure 3.4 shows randomized results that indicate that bootstrapping SSPs is problematic.”*

### **Terminology: Equations within quotes**

*Inside a quotation, [ [double bracketed Equation numbers] ] indicate that equation exists within this dissertation’s set of numbered equations. The actual quote had a different number that is the quoted author’s publication. The Equation number for the same equation found elsewhere in this dissertation, is inside the double bracket inside the quotation.*



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A BASIS TO REDUCE EMISSIONS AND IMPROVE IMAGES**

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# CHAPTER 1

## EXECUTIVE SUMMARY

Many methods to improve Underwater Acoustic (UWA) communications (aComms) or imaging use active sensing during data collection. The objective of this dissertation is to provide to the UWA community a set of *purely* statistical tools to provide *passive* forecasting and control input in order to reduce wavefront distortion. Used in other disciplines, these methods are data-matched to automated control systems that ameliorate wavefront distortion [7–10], [11, slide 34]. UWA has different “adaptive acoustics” methods that switch protocols. Another method, Matched-Field Processing (MFP) mostly does for aComms what *Adaptive Optics* (AO) does in the atmosphere and for satcom. The type of MFP this dissertation uses was derived for cases where the range of remote sensing or communications is shorter than a kilometer (km). Successful MFP allows reception of signals at lower transmit power.

This dissertation provides the UWA distortion for aComms or remote sensing using measured and verified SSPs for 132 different locations in the Atlantic Ocean near the Caribbean, digitized from a NAVO Atlas [3], as well as 64 Sound-Speed Profiles (SSPs) in two areas in the littorals, Saint Andrew Bay in Panama City and a marina in Dania Beach connected to Port Everglades. The phase variance is proportional to range, coherence length, the square of the acoustic frequency, as well as these distortion values (represented by *Background* Structure Functions [12, App B], BSFs). These statistical structure functions [2, 4, 13, 14], [15, p 10] are only similar to covariances under Locally Homogeneous and Isotropic (LHI) conditions [12, p 333]. Therefore, the phase variance for the deep-sea locations as an ensemble throughout the Atlantic, and the more localized ensembles in the Bay and the Dania Beach marina used by



Florida Atlantic University (FAU) are available for different spatial periods (depth differences) and locations. Since these data are in the format used by the commercial systems (such as Wavetrain<sup>®</sup> [11]) this work provides a basis for using such forecast and control software to provide phase parameters for transmission arrays. While equalizing multipath is beyond the scope of this dissertation, the methods used to provide gain using multipath (including bottom or surface “reflections”) are similar to those used to similarly constructively interfere micropaths within the same or similar “tubes.”

“Fluctuations due to sound interaction with ill-characterized interfaces such as the bottom sediment prevent *a priori* path gain phases to be estimated with reasonable accuracy at the frequencies ...” [16, p 993]

Equation 1.1 provides the deterministic part of the “path response” as a function of the delay,  $\tau_{k,p}(t)$ , at time,  $t$ , for path,  $p$ , and receiver array element,  $k$ .

$$f_{k,p}(t) = s_{k,p}(t)\delta[\tau - \tau_{k,p}(t)] \tag{1.1}$$

“The adaptation of [deterministic response, Equation 1.1] at the receiver is based on a static or dynamic model of the environment that includes, among other parameters, bottom bathymetry, sound-speed profiles and source/receiver positions. It is clear that such detailed information is generally unavailable when surveying a new area in the ocean, but assuming its existence may be realistic, e.g., when a permanent underwater base-station receives transmissions from mobile sources. Not only is it possible for the base station to have fairly detailed knowledge about its (essentially fixed) environment, but it may also have the computational resources to exploit it effectively.” [16, p 993]

By better equalizing the delays, which can allow more microrays to be better summed with the direct ray, systems such as equalizers can improve communications and remote sensing. Where coherence is sufficient (Appendix B) reduction of transmission power has little effect on image resolution. Since Signal to Noise and Inter-symbol Interference Ratio (SNIR) is sensitive to high transmit power, producing

the onset of clipping in extreme cases, then when lower power is sufficient, SNIR is less sensitive to changes in power. Before getting into SNIR details, there are some considerations of scale for overall Signal to Noise Ratios (SNRs):

“In contrast, large-scale variations influence the SNR through its local average, causing it to vary over longer periods of time. As such, they are meaningful for the analysis of top-level system functions such as power allocation and the assessment of outage probabilities and statistical coverage.” [5]

There are several sources of interference including biological acoustics that also produce the signal processing form of symbol interference. However, one source of interference that relates to the transmitted power is the “resolution” or contrast of the change in signal that, when detected as part of a small group of changes, indicates a particular symbol. Extreme error can lead to clipping and a signal effect called quantization noise. Since acoustic pressure intensities collected over time represent symbols, they are more sensitive to those power changes when the overall transmitted power is lower. Properly equalized signals can combat much of this noise by combining signals from many microrays to increase the SNIR, provided the constructive “interference” exceeds the destructive interference. Existing equalized SNIR, unaltered (untuned) for the refraction effects this dissertations BSF method measures, are tabulated in the Dania Beach Marina for combinations of “using three, four or five receivers” [10].

The evolution of different methods are crucial to the jargon differences and similarities and the difficulty of different disciplines to communicate concepts with each other. Some of these concepts are discussed in this dissertation, including trade-offs between phase screens versus path-integrals, match-field/time-inverse-mirror versus ray-tracing, structure functions [15, p 10] versus spatial power spectra [12, App B, p 526], [15, p 17], and deterministic versus stochastic models. Historically, multi-phase screen analysis methods for UWA were developed in the 1950’s. Due to its

straighter propagation, the analysis of electrodynamic wavefront distortion in atmosphere took a different “path.” Whereas, over fifty years ago UWA sensibly chose the full path integration method because (computers did not exist, and) path-integrals match the sometimes incredibly curvilinear trajectories for long-range transmissions in the ocean. The path length differences of different ‘microrays’ produce signal delay differences that occur at the receiver.

Inter-Symbol Interference (ISI) can occur for different microrays, our main path difference concern in this work, and especially for multi-path. The latter is a well studied problem in optics, RF, and UWA. This form of ISI, combined with ISI from acoustic channel variations during message transmission, are major challenges to successful underwater Acoustic Communications (aComms). Typical aComms frequencies of 10-20 kHz, and especially the *high-frequency* modem downlink 65 kHz, and uplink 262-375 kHz [8, 10], [17, Table 9], which are within the target passbands under consideration for this dissertation, have “small” wavelengths, 15-7.5 cm, 2.3 cm, and 5.7-4 mm respectively. Interference patterns in the propagating acoustic field due to these wavelengths are “comparable, e.g., to the uncertainties in positioning of projectors [transmitters] and hydrophones.” [16]

This dissertation follows the optics evolution through development of computers in the 1970s, where phase screens break up the propagation path into profile planes characterized by measured sound speed profiles (SSPs) that can forecast the phase distortion that receiver arrays can sense. From 1950-1980 the optical and radio “frequency” (RF) engineers could not have known how their discretized system would be so amenable to the rise of computing machines.

The sum of delays across any *profile plane*, perpendicular to the propagation, builds up a phase screen representation of the wavefront distortion, a phase function laterally, in  $y$ , and depth,  $z$ ,  $\phi_i(y, z)$  for the  $i_{\text{th}}$  phase screen. Classical UWA taught the phase screen idea, but uses a full trajectory phase to analyze the delay *measured*

*at the receiver*, where each delay has a different Doppler to identify it. The use of the BSF requires that the phase screen definition is on a profile plane that segments the full trajectory. If two phase screens are used they would represent two segments, near, and far, with an interface between the two characterizations between the phase screens. A single phase screen, Figure 2.2 can model a short-range transmission, perhaps less than 200 m.

During this history some of the industrial infrastructure built by other disciplines were serendipitously designed to provide results based on the discrete nature of multiple phase screens. The development of computers enhanced the phase screen method's utility and accuracy. This discrete nature appears to be adequate for sub-kilometer propagation or image sensing. The structure functions for both methods were designed by Tatarski (spelled Tatarskii some references) and translated into English for a 1961 textbook on acoustic propagation and other effects [15, 10]. These are the BSFs displayed by the author in his OCEANS 22 presentation and paper [18].

A second result of using the structure functions is to have a metric for the quantity of wavefront distortion that is tip/tilt (laterally shifted). This has important results for image correction to remove lensing/breathing/ducting from a time-sequence of images. Different algorithms work differently under different distortion distributions. Knowledge of the statistical distribution of distortion across a few Zernike modes [13, 19] (piston, tip/tilt, defocus, astigmatism, coma, ...) can determine the most effective image processing algorithms to employ.

It is crucially important to understand the bifurcation of historical development in order to utilize existing, proven, industrial software linked to hardware that runs such seemingly different applications as satellite communications and their underlying electro-optical systems. Those wavefront sensing systems are established from the same Tatarskii method [15, 10] using structure functions, developed in the 1950's, as UWA propagation structure functions with more than dozens of Degrees of Freedom

(DOFs). Alternatively, *background* structure functions (BSFs) model the wavefront distortion at a particular station,  $i$ , along the propagation, along the  $x$ -axis, with a maximum of a few DOFs.

UWA understandably developed intricate integro-differential expressions to deal with the incredible kilometer sized vertical bounces that acoustic trajectories undergo in the deep sea. Using the atmospheric methods for UWA would require some intricate modeling that would require a brilliant solution, or a brute force system. The brute force system requires numerous multiple phase screens and curvilinear trajectories with a tight resolution. This is meant to match the existing utility of ray tracing, eigen-state, and other method the UWA community developed. However, there are other UWA wavefront propagation requirements that the industrial method can model. This is the story of fitting those methods to UWA for littoral or shallow sea systems, *starting with the single phase screen system* to model a short range.

As with many multi-disciplinary investigations, jargon for the same math varies by discipline. By breaking through this barrier the use of systems from other disciplines becomes available. The languages of atmospheric propagation for *satcom* and remote sensing, mostly within the field of adaptive optics (AO), grew apart from that for UWA in the past 70 years of their separate use. They are both mathematics-based languages for scalar solutions of a Helmholtz wave equation, both usually using the parabolic and paraxial approximations. However, many subjects within these disciplines use entirely different language for the same math. For example, before the advent of acoustic arrays using beamforming, UWA sensors were non-imaging collectors of sound that arrives at different times for which the analyst seeks to discern the possible physically separated trajectories for the different sound signals arriving at solitary sensors.

Another difference is that these phase screen systems are primarily purely stochastic systems apart from using the distance between stations as a solitary DOF,  $x_{n+1} - x_n$

to provide a phase change for the entire phase screen (or segment of the profile plane thereof). Solitary phase screens, upon which the BSF definition takes place, often omit the “thickness” that the phase screen’s Dirac delta function along the x-axis represents, because that is the same as the range  $x_{\text{Rx}}$ . These differences alone can cause some confusion. Further, many subjects and variable names seem quite similar in spite of having nearly opposite meanings. An unpublished set of tables, produced as a class project, may be available to the enterprising future author who might want to provide a translation between these languages. Selected examples of jargon conflicts are discussed herein where necessary.

Wavefront distortion can affect the performance of aComms and imaging systems. A measure of wavefront distortion, the *background* structure functions are functions of sound speed profiles (SSPs) that are functions of depth. Classical UWA structure functions are a combination of deterministic and statistical (sets of microrays) in an attempt to keep track of sensed signal delays for the same critical ray signal. Long term, breaking the propagation up into stations of BSFs can make established commercial software available to forecast phase statistics for coherent systems and eventually to reduce wavefront distortion. Historically, UWA used a suite of methods (see [4]) that include path-integral, ray tracing [20, p 12], and modal methods [1, 21] replaced the initial methods for statistical acoustics developed in the latter half of the last century. Methods such as ray tracing have limits, as do the statistical methods. For example,

“Beam tracing tools, such as Bellhop, use ray theory to provide an accurate deterministic picture of a UWA channel for a given geometry and signal frequency, but they do not take into account random channel variation.” [5]

Developed before computers to deal with features such as large bounce distances, methods of Under-Water (UW) acoustics provide an amalgamation of deterministic and statistical expressions to help estimate Doppler and delay for UW acoustics by

integrating the classical structure functions from the source to the receiver. In the interim years, non-acoustic communications industries used the older methods with algorithms and hardware that alleviated some of the old shortcomings in order to forecast communication performance. For acoustics, the use of databases of BSFs and SSPs recently helped simulate proof-of-concept forecasting of aComms operating envelopes. These purely statistical BSFs have less than a quarter of the DOFs in classical UW structure functions. Symmetries also drastically reduce the required background DOFs. For decades industry used these purely statistical structure functions to estimate wavefront wander, lensing (ducting), and fade.

Statistical methods are apparently a recent development. As of 2013 the state of the art was nascent:

“Statistical modeling of small-scale phenomena is a subject of ongoing research, which points to different types of fading, and no consensus exists yet on this topic. Modeling of large-scale phenomena has also been addressed only to a very limited extent ...” [5]

Commercial methods to estimate communications (comms) fade assume the medium has BSFs that are sufficiently symmetric in order to simplify the calculations. While these metrics can require re-calculation at different stations along the propagation path, extra phase screens are compatible with the evolution of computer technology during the past half century. Conditions of acoustical symmetry and invariance allow simplification of the acoustical calculations in most of the deep-water ocean and in littorals. Results from shallow-sea measurements show that background BSFs in the littorals validate the simplified calculations for sufficiently symmetric sound speed fields. However, the complex calculations that arise from removing the simplifications are tractable with recent algorithms. Adaptive wavefront methods can use BSFs with arrays to guide the wavefront through less lossy paths. Nevertheless, even the value of the ability to forecast phase and throughput statistics, in locations where measurements or estimates of sound speed fields exist, is of adequate value

to provide a foundation for the future use of these purely statistical methods. This dissertation is meant to be this foundation for the UWA-BSF-phase-screen method. In the AO applications industry uses these methods to drastically reduce the amount of projected power required to attain the same sensed contrast and information at the same range as a non-AO system, while increasing the imaging and packet detection resolutions. Therefore it is a worthy goal to bring those advances as well as novel wavefront sensing systems into the UWA mainstream.

The main technical result for this dissertation is characterization of phase in order to avoid losing lock in equalizers for high-bit-rate aComms. The introduction itself shows that, except for a constant ( $0.25\pi^{-2}$ ) the phase structure function (PBSF) is *identical* to the BSF, a concept alluded to in the literature and identified in detail in this dissertation. This analytical indulgence of integro-differential calculus is necessary to provide a validated expression for the phase variance. The publication of a part of this dissertation, Chapter 4, an analysis the Gouy phase anomaly for the main acoustic channel in the open ocean, addresses due to the importance of errors in phase estimates [22].

The author's M.S. thesis averaged the phase from a probe beam on a vibrating target [23]. That work is only a spatial effect. However, it is related to arrays of acoustic receivers discussed in the subsection just before §2.6 on page 66 and also in Appendix B including Signal Excess similarities in Table B.1 on page 180.

During this last year, the author received two United States patents that were investigations inspired by acoustics classes and the projects leading to this dissertation:

US 11431421, "Caustic Expander & Local Waveguide to Improve Acoustic Comms"  
30Aug22

US 11653125 "Method [to Collect] Field-Based Data to Reduce Collected Data Error" 16May23